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DETERMINATION OF THE ANGULAR COORDINATES OF ASTRONOMICAL OBJECTS THROUGH OPTIC-ELECTRONICAL MEASURING SYSTEMS

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Abstract

Among the methods for determination of the angular coordinates of astronomical objects, particularly topic is the Turner method based on static processing of data information for star number of three and more.

With regard to optical electronic apparatuses used in astronomical practice which high resolution and vision field lower than one degree feature the methods for angular coordinates determination with minimum number of supporting (catalogical) stars become topical. In such conditions it is necessary to re-examine the attitude to orthogonal methods, whose accuracy can be increased through statistical analysis.

The calculation of the ideal coordinates and the equatorial coordinates of an astronomical object is shown as well as the possible errors in their determination.

A priori it is supposed that the orthogonality method is more effective because of the fact that the possibility for appearance of two stars in the vision field is greater than for three ones.

Among the methods for determination of the angular coordinates of astronomical objects, particularly topical is the Turner method based on statical processing of data information for three and more supporting stars number $k \geq 3$ and the method of four constant (orthogonal) ones using two supporting points [1 ,2, 3].

In relation with the widely spread optic-electronical measuring systems in astronomical practice, which have high-resolution ability and relatively a narrow vision field- smaller than one degree [6], the methods for determining the angular coordinates arises with minimum number of points of support (catalog) stars. Become topical in these conditions, the attitude to the orthogonal methods has to be reconsidered, the accuracy of which can be increased by way of statistical arrangement in $k > 2$.

The influence of differential effects on optical systems with a narrow vision field is insignificant.

A method for determination of angular coordinates of astronomical objects, spreading on stars number $k > 2$ and calculation of objects coordinates by the least square method is presented.

The correlation between the ideal and the measured coordinates is expressed by formulas [1,3]:

$$(1) \qquad \begin{aligned} \zeta_1 &= ax_i + by_i + c \\ \eta_i &= -bx_i + ay_i + f \end{aligned} \qquad \text{if } \ i = 1, k,$$

where: x, y – calculated coordinates of supporting stars;
ξ, η – ideal coordinates of supporting stars.

The expression (1) can be also wretten in the following way:

$$(2) \qquad v_{1,i} = x_{1,i}a_1 + x_{2,i}a_2 + 1a_3 + 0a + \xi_{1,i}$$

or in matrix form:

$$(3) \qquad \vec{v} = \overrightarrow{Xa} + \vec{\xi}$$

where: X is a matrix made by rows

$$\vec{x}_{1,i}\{\vec{x}_{1,i}, \vec{x}_{2,i}, 1, 0\}$$

(4) and

$$\vec{x}_{2,i}\{\vec{x}_{2,i} - \vec{x}_{1,i}, 1, 0\}$$

and calculating the equations

$$(5) \qquad Q = X'X, K = Q^{-1} \quad , \quad \vec{L} = X'\vec{\xi}, \vec{a} = -K\vec{L} \quad ,$$

where: $\vec{a}$ – vector evaluating the constant staff;

Q – a matrix with normal equations as follows:

$$(6) \qquad \begin{matrix} [x_1^2]+[x_2^2] & 0 & [x_1] & [x_2] \\ 0 & [x_1^2]+[x_2^2] & [x_2] & -[x_1] \\ [x_1] & [x_2] & k & 0 \\ [x_2] & -[x_1] & 0 & k \end{matrix}$$

The cell method for matrix calculation [10] with regards to the symmetry and other characters of the matrix Q applying, it allows to find the most suitable form for passing from the calculated coordinates to the matrix K elements, forming and transporting the matrix X, and forming and rotating the matrix Q. The matrix K is as follows:

$$(7) \qquad K = Q^{-1} = (X'X)^{-1} = \frac{1}{W},$$

where: $R = [x_1^2] + [x_2^2]$;

$$W = kR - [x_1^2] - [x_2^2]$$

The elements of the vector $\vec{L}$, have the following values:

$$\text{(8)} \qquad \vec{L} = X'\vec{\xi} = \begin{vmatrix} [x_1\xi_1] + [x_2\xi_2] \\ [x_2\xi_1] + [x_1\xi_2] \\ [\xi_1] \\ [\xi_2] \end{vmatrix}$$

For ideal coordinates of the astronomical object, the following expression holds:

$$\vec{a} = -KL \;,$$

$$\text{(9)} \qquad \xi_1 = \vec{x}_1, \vec{a} \;,$$

$$\xi_2 = \vec{x}_2, \vec{a} \;,$$

and its equatorial coordinates' estimates at known formula [1]:

$$\text{(10)} \qquad \alpha = \operatorname{arctg}\left(\frac{\xi_1}{\cos D - \xi_2 \sin D} \right) + A$$

$$\text{(11)} \qquad \delta = \operatorname{arctg}\left[\frac{(\sin D + \xi_2 \cos D)\cos(\alpha - A)}{\cos D - \xi_2 \sin D} \right],$$

where A and D are the equatorial coordinates in the stuff centre.

The errors in obtaining the coordinates of the object Δ_α and Δ_δ are collected from the error in the reduction Δ_ξ and the error of the calculated coordinates of the object Δ_x.

$$\text{(12)} \qquad \dot{\Delta}_\alpha = \dot{\Delta}_{\xi_1} + M\dot{\Delta}_{x_1} \qquad \text{and} \qquad \dot{\Delta}_\delta = \dot{\Delta}_{\xi_2} + M\dot{\Delta}_{x2}$$

Here, the scale multiplier: $M = \sqrt{a_1^2 + a_2^2}$ [1].

The reduction error in solving a system of 2k equations of four unknowns by the least square method is determined by dispersion [11]:

$$\sigma^2\left(\dot{\Delta}_{\xi_1}\right) = \frac{\left|v_1^2\right|}{2(k-2)} P_1 \quad , \quad \sigma^2\left(\dot{\Delta}_{\xi_2}\right) = \frac{\left|v_2^2\right|}{2(k-2)} P_2 \tag{13}$$

where the weight coefficients P_1 and P_2 are determined by formulas [11]:

$$P_1 = \sum \left(\frac{\partial \xi_1}{\partial a_j} \right)^2 K_j + 2 \sum \left(\frac{\partial \xi_1}{\partial a_i} \right) \left(\frac{\partial \xi_1}{\partial a_j} \right) K_i \; ; \tag{14}$$

$$P_2 = \sum \left(\frac{\partial \xi_2}{\partial a_j} \right)^2 K_j + 2 \sum \left(\frac{\partial \xi_2}{\partial a_i} \right) \left(\frac{\partial \xi_2}{\partial a_j} \right) K_i \; . \tag{15}$$

Having attention to $\frac{\partial \xi}{\partial a} = \vec{x}$, P_1 and P_2 can be presented in square form suitable for the vectors $\vec{x}_1$ and $\vec{x}_2$ and the matrix K [8]:

$$P_1 = \overrightarrow{x_1}' K \overrightarrow{x_1} \qquad \text{and} \qquad P_2 = x'_2 K x_2 \tag{16}$$

The final formula accuracy estimating is the accuracy:

$$\tilde{\sigma}\left(\dot{\Delta}_\alpha\right) = \sqrt{\frac{\left|v_1^2\right|}{1(k-2)} P_1 + M^2 \sigma^2\left(\dot{\Delta}_{x_1}\right) \cos D} \tag{17}$$

$$(18) \qquad \tilde{\sigma}(\dot{\Delta}\delta)=\sqrt{\frac{[v_2^2]}{l(k-2)}P_2+M^2\sigma^2(\dot{\Delta}_{x_2})}$$

In conclusion it has to be marked the question for comparison of the effectiveness of that and the other methods can be a topic of special investigation but apriori it is supposed the orthogonality method is more effective because of the fact that the possibility for appearance in the two stars field of vision is bigger than of the three ones.

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ОПРЕДЕЛЯНЕ НА ЪГЛОВИТЕ КООРДИНАТИ НА АСТРОНОМИЧНИ ОБЕКТИ ЧРЕЗ ИЗПОЛЗВАНЕ НА ОПТИКО-ЕЛЕКТРОННИ ИЗМЕРВАТЕЛНИ СИСТЕМИ

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Резюме

Използването в астрономичната практика на оптико-електронни системи с висока проницателна способност и сравнително тясно зрително поле мотивира актуалността на методите за определяне на ъгловите координати на минимален брой опорни (каталожни) звезди.

Предложен е метод за определяне на ъгловите координати на астрономични обекти, разпространени на брой звезди $k > 2$ и изчисляване на координатите на обекти по метода на най-малките квадрати.

В заключение е отбелязано, че проблемът за сравняване на ефективността на този и други методи трябва да бъде предмет на специално изследване, но априорно може да се предположи, че за оптико-електронни системи ортогоналният метод е по-ефективен поради факта, че вероятността за поява в зрителното поле на две звезди е по-голямо, отколкото на три.